Talbot bands and temporal processing of optical signals

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Talbot’s bands can be observed in one of the diffraction orders of a grating spectrometer with a partially phase-shifted input beam. This experiment has long been considered as a curiosity. Here, we look at it from a new perspective: we discuss the Talbot band experiment as a combination of two kinds of dispersion, refractive and diffractive. This may be of interest for various tasks of temporal optical signal processing such as dispersion management and the shaping of ps/fs-pulses. For this purpose, we demonstrate the use of micro machined optical elements to alter the performance of the classical Talbot band setup. A theoretical description and experimental results are presented. [DOI: 10.2971/jeos.2006.06001]

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1 Introduction

H. F. Talbot is widely recognised for his work on the self-imaging effect published in 1836 [1]. On the other hand, another experiment described by Talbot in 1837 [2] on the formation of interference fringes in white light has gained relatively little attention so far. In that experiment, Talbot observed the formation of interference fringes (“Talbot bands”) in one of the two first diffraction orders of a grating spectrometer upon inserting a piece of glass of appropriate thickness halfway into the illuminating beam Figure 1. This observation is somewhat surprising, in particular, the fact that the bands occur only in the +1st order (notation as defined in the figure) and not in the -1st order. Fringes are shown in Figure 1.

The occurrence of the interference fringes in the +1st order can be explained qualitatively in the temporal domain. For this, it is suitable to view broadband light as consisting of short pulses. For our brief review of the experiment, we will follow the description presented by Lohmann [3], however, we would like to mention two other articles by Parker Givens [4] and by Benkő, Hilbert and Bor [5].

We assume that the aperture diameter of the setup shown in Figures 1 and 2 is \( W = Np \) where \( p \) is the period of the grating and \( N \) is the number of periods in the grating. Furthermore, we assume that the glass plate covers half of the aperture. Intuitively, it is clear that the experimental observation described in Section 1, Figures 1 and 2, is caused by the asymmetry between the optical paths in the +1st and the -1st order. To understand this, we look at three snapshots taken at three different times \( t_1, t_2 \) and \( t_3 \) and consider two pulses: one travelling through the upper half of the aperture, i.e., through the glass plate, one travelling through the lower half Figure 3. At \( t = t_1 \), the two pulses are located at the same longitudinal position just before the glass plate. A short time later, at \( t = t_2 \) the upper pulse has passed through the glass plate. It now trails the beam in the lower half which has travelled through air. The optical path length difference is \( \Delta n l \). Here, \( \Delta n = n - 1 \) is the index difference between glass and air as a function of the temporal frequency, \( d \) is the thickness of the glass plate. We denote the temporal delay between the lower and the upper beam as

\[
\Delta t = \frac{Dn d}{c}
\]  

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If the thickness is $\lambda_0/2$ around a central wavelength $\lambda_0=550$ nm corresponds to pulse duration $\Delta t \approx 6$ fs. Such short pulses were first demonstrated in [6] and pulses of typically less than 30 fs are nowadays generated by commercial laser systems.
When a pulse passes through an optical element (a phase plate or a grating) or a whole system, it experiences dispersion, i.e., the occurring time delay varies with frequency. In the case of linear devices we can use the concept of the temporal transfer function, here denoted by \( \tilde{H}(\nu) \) to establish the relationship between input and output spectra:

\[
\tilde{S}_{\text{out}}(\nu) = \tilde{S}_{\text{in}}(\nu)\tilde{H}(\nu)
\]

(10)

Assuming no absorption and neglecting polarisation effects, we can write for the transfer function \( \tilde{H}(\nu) \):

\[
\tilde{H}(\nu) = \exp \left[ 2\pi i t(\nu)\nu \right]
\]

(11)

In the qualitative explanation of Section 1, the time delay was considered to be constant. In order to take dispersion into account, we write for the refractive time delay due to the phase plate, \( t_r(\nu) \), as a function of the frequency:

\[
t_r(\nu) = \frac{\Delta n(\nu) d}{c}
\]

(12)

For a glass plate with normal (anomalous) dispersion, the index of refraction increases (decreases) with \( \nu \), i.e., \( dn(\nu)/d\nu > 0 \) (\( dn(\nu)/d\nu < 0 \)) and hence

\[
dt_r(\nu)/d\nu \begin{cases} < 0 & \text{anomalous} \\ > 0 & \text{normal} \end{cases}
\]

(13)

And for the time delay \( t_d(\nu) \) due the grating it is

\[
t_d(\nu) = -m \left( \frac{N}{2} \right) \frac{c}{\nu}
\]

(14)

from which we obtain:

\[
dt_d(\nu)/d\nu = m \left( \frac{N}{2} \right) \frac{c}{\nu^2}
\]

(15)

Notice, that for complementary orders, for example, the +1st and -1st order, we have opposite dispersive behaviour:

\[
\frac{dt_d(\nu)}{d\nu} \bigg|_{m=1} = -\frac{dt_d(\nu)}{d\nu} \bigg|_{m=-1}
\]

(16)

Although, of course, dispersion is due to diffraction in both cases, one may use the combination of two complementary orders for compensation purposes. This is widely done for achromatization of all-diffractive systems [7] and for pulse shaping devices using double diffraction [8, 9]. The point of view of considering the Talbot bands as a combination of refraction and diffraction was discussed in detail in Ref. [3]. In particular, various cases of combining different element have been discussed. We now want to return to the case of the Talbot bands and see how the combination of different refractive and diffraction characteristics can be exploited. In the next section, we calculate the intensity of the Talbot bands.

3 WAVE-OPTICAL ANALYSIS

We calculate the far-field pattern dependent on the observation angle \( \alpha \). The temporal field travelling along that direction consists of a sequence of pulses (see Figure 4):

\[
S(t, \alpha) = \sum_{m=0}^{N/2-1} S_0(t - mt) + \sum_{m=N/2}^{N/2-1} S_0(t - mt + t_r)
\]

(17)

\[
\tilde{S}_0(t) \text{ is the spectrum of the individual pulse according to Eq. (6). } \tilde{H}(\nu, \alpha) \text{ is the transfer function of the optical setup along the direction of observation. The left side of this equation states that the intensity is given by integrating over the intensities in the pulse sequence. The right side follows from Eq. (10) and Parseval’s theorem. We can further split up the transfer function into two terms:}

\[
\tilde{H}(\nu, \alpha) = \sum_{m=0}^{N/2-1} \exp \left( 2\pi i m t_r \right) + \sum_{m=0}^{N/2-1} \exp \left( 2\pi i \left( m + \frac{N}{2} \right) t - t_r \right)
\]

(19)

\[
= \left\{ 1 + \exp \left[ 2\pi i (Nt/2 - t_r) \right] \right\} \cdot \sum_{m=0}^{N/2-1} \exp \left( 2\pi i m t \right)
\]

\[
\tilde{F}(\nu, \alpha) \cdot \tilde{G}(\nu, \alpha)
\]

\[
\text{and obtain the following expression for the intensity (by dropping a factor of 2):}
\]

\[
I(\alpha) = \int \left| \tilde{S}_0(\nu) \right|^2 \left| \tilde{F}(\nu, \alpha) \right|^2 \left| \tilde{G}(\nu, \alpha) \right|^2 d\nu
\]

(20)

The first term in the integral, \( \left| \tilde{S}_0(\nu) \right|^2 \) is the power spectrum of the optical pulse. The third term,
\[ |G(v)|^2 = \left| \sum_{m=0}^{N/2-1} \exp(2\pi imvt) \right|^2 = \frac{\sin^2(\pi(N/2)vt)}{\sin^2(\pi vt)} \]  

(21)

is the typical term that one obtains for multiple beam interference and describes the different diffraction orders. The modulation of the diffraction orders, i.e., the structure of the Talbot bands, is given by the second term, \(|\tilde{F}(v)|^2\). For the classical Talbot band experiment, one obtains:

\[ |\tilde{F}(v)|^2 = 2 [1 + \cos(2\pi \phi)] \]  

(22)

with

\[ \phi = \frac{\pi}{2} N t - \frac{\pi}{2} \nu p \sin \alpha / c - \Delta nd \nu / c \]  

(23)

The sinusoidal modulation is typical for two-beam interferometers. Its occurrence is not surprising here, since the incoming beam is subdivided into two beamlets by the glass plate. Since both share the same aperture, we may call the conventional Talbot band setup an “in-line two-beam interferometes. Its occurrence is not surprising here, since the incoming beam is subdivided into two beamlets by the glass plate.

First, we generalise our theory. We assume that instead of a simple glass plate a structured phase element is inserted into the aperture, we can achieve a more complex temporal modulation across the incoming beam. We demonstrate this for two cases denoted as experiments 2 and 3.

In experiment 2, the phase object consisted of a linear staircase pose it consists of \(K\) discrete steps of equal width \(w\), i.e., \(MK = N\). Then we can re-write Eq. (17) as

\[ S(t, \alpha) = \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} S_0 \left[ t - (m + kM) t + t_r^{(k)} \right] \]  

(27)

\(t_r^{(k)}\) is the time delay generated by the k-th interval. For the classical Talbot band experiment, it is \(K = 2\), \(M = N/2\) and \(t_r^{(k)} = kt_r\). With Eq. (17), we can easily find:

\[ \tilde{H}(v, \alpha) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} \exp \left[ 2\pi iv \left( (m + kM) t + t_r^{(k)} \right) \right] \]  

(28)

\[ = \sum_{k=0}^{K-1} \exp \left[ 2\pi iv \left( kMt - t_r^{(k)} \right) \right] \cdot \sum_{m=0}^{M-1} \exp (2\pi imvt) \]

\[ = \tilde{F}(v, \alpha) \cdot \tilde{G}(v, \alpha) \]

In experiment 2, the phase object consisted of a linear staircase as shown in Figure 6. Width and height of the steps have to be chosen appropriately: since the aperture is subdivided into more and narrower steps, the step height has to be reduced. The condition is derived from \(\Delta nd' = \sin \alpha\) from which follows (analog to Eq. (4)):

\[ d' = M\lambda / \Delta n = (W/K)\lambda / \Delta n = d / K \]  

(29)

Note that this is the same condition like Eq. (4). Figure 5 shows a measured intensity plot for the band structure of Figure 3.
The intensity distribution of the fringe pattern can be calculated with the theory presented earlier. The linear staircase generates $K$ beamlets with multiple delays: $t_r^{(k)} = k\tau$. We obtain:

$$S(t, \alpha) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} S_0 [t - (m + kM) t + kt_r]$$  \hspace{1cm} (30)

The intensity of the modulation varies with:

$$|\tilde{F}(\nu, \alpha)|^2 = \left| \sum_{k=0}^{K-1} \exp [2\pi i \nu (M\tau - t_r)] \right|^2 = \frac{\sin^2 (\pi K\phi)}{\sin^2 (\pi\phi)}$$  \hspace{1cm} (31)

For the linear staircase, we thus obtain a modulation of the Talbot bands as it is typical for a multiple beam interferometer with $K$ beams. Thus the peaks appear sharper than for the classical Talbot band experiment (see Figure 7).

In experiment 3, a simple binary phase grating is used (Figure 8). The period $2w$ of this element is rather coarse, so that diffraction can be neglected. Spatially, the structure subdivides the incoming beam into $K$ beamlets as in experiment 2 with a similar reduction of the step height. Temporally, however, only a single time delay occurs as in experiment 1. Hence that pattern is also sinusoidal. However, the smaller step height results in a shorter time delay than in experiment 1. Therefore in the $\nu$-domain a fringe period is observed whose period is increased (Figure 9). In the angular domain, the period is $p'_\tau = K\lambda/W$. It should be noted, that in the case of an object with lateral symmetry, one observes a fringe pattern also in the -1st order since pairs of pulses also form for direction $-\alpha$.

5 CONCLUSION AND OUTLOOK

So far, the Talbot band experiment has been mostly considered as a curiosity and as an example of the role of temporal aspects in optical interferometry. Here, we have considered a different point of view: the Talbot band experiment suggests that a combination of two different kinds of dispersion, diffractive and refractive, yields surprising results. That may be inspiring for the so-called “dispersion management” which is an important issue for temporal signal processing at ultra high speed. Yet more general effects might emerge if also a third tool, reflective dispersion, is used in a mixed fashion. That idea becomes clear by considering in Figure 10, a reflective implementation of the classical Talbot band experiment of Fig. 1. Here, a micro machined mirror causes the delay between the left and the right half of the input aperture.
The use of micromachined optical elements with depths ranging between 10 and 1000 µm allows one to implement optical delays from the fs- up to the ps-range. This may be used to implement an optical pulse shaper based on the Talbot band experiment beside well-known setups such as the Treacy interferometer [8] and the widely used far-field grating interferometer, GI [9]. Far-field GI and the Talbot band setup are shown in a combined drawing in Figure 11. In the GI, a first grating, $G_1$, forms the spectrum in the filter plane $F$ (denoted by the coordinate $\nu$). A mask is placed in this plane as a spectral filter. A second grating, $G_2$, recombines the different wavelength components. In the Talbot experiment, the phase element is positioned in the Fourier-conjugate plane near the first grating. In comparison one may say that in the GI-setup filtering is performed in the frequency domain, in the Talbot band setup we influence directly the temporal shape of the signal. A point-like filter, for example, in the $\nu$-domain corresponds to a prismatic structure (the linear staircase) in the temporal domain.

**References**


