This paper describes a novel approach for realizing femtosecond optical frequency comb (FOFC)-based length measurement. This approach is based on the analogy between the phase unwrapping problem and the integer ambiguity problem. Because the conventional synthetic wavelength method can solve the former, we investigated the possibility of using a synthetic adjacent pulse repetition interval length method to solve the latter. The results of theoretical analyses and numerical investigations show the feasibility of the proposed method. Our results should contribute toward the further development of FOFC-based length measurement methods.

Keywords: Instrumentation, measurement, optical frequency comb, interferometry, metrology

1 INTRODUCTION

Recently, numerous studies [1, 2] have focused on femtosecond optical frequency comb (FOFC)-based length measurement because this approach provides a highly accurate frequency reference for measurements. Length measurement using an FOFC source has distinct measurement characteristics [3, 4] and applications [5]–[7]. Many special techniques [8]–[11] have been developed to exploit these distinct characteristics (especially the temporal coherence) of FOFC light. A research group at the National Institute of Advanced Industrial Science and Technology has extensively studied length and length-related measurements using FOFC [12]–[16]. In particular, Minoshima and Matsumoto have pioneered the use of FOFC-based approaches for measuring large lengths with their original demonstration of the measurement of a 240-m distance [12].

Figure 1 shows a schematic of the interference fringes obtained by a conventional Michelson interferometer with an FOFC light source. The correlation (autocorrelation and cross correlation) functions between two pulse trains afford two length scales: the central wavelength \( \lambda_{cen} \) and the adjacent pulse repetition interval length (APRIL) \( c \times T_R \), where \( c \) is the speed of light in vacuum and \( T_R \) the pulse repetition period. When seeing one correlation function, the distance between the two peaks of the interference fringes can be understood to be half the central wavelength of FOFC. In general, the size of the central wavelength can be continuously used as the length scale within this range. When looking at the whole correlation functions, the distance between the two peaks of the envelope of the interference fringes can be understood to be half the APRIL of FOFC. In general, the size of the APRIL is of the order of meters. Furthermore, the existence of the APRIL is discrete. Below, we discuss how to perform length measurement by using the central wavelength and/or APRIL.

First, we consider the possibility of using the central wavelength of an FOFC as the length scale. Because the range of one correlation function is too short (of the order of micrometers) to cover a long distance (for example, of the order of meters), it is impossible to simply use the central wavelength as a length scale in an ordinary Michelson interferometer. We can extend the range to observe the correlation function by using multiple reference mirrors. However, because the target could be as much as around 10000 times larger than the range of one correlation function, the feasibility of this approach is low.

A single-wavelength helium–neon (He–Ne) laser is used as a length standard. The use of an FOFC to produce a single wavelength appears promising because we can use a He–Ne laser interferometer by simply changing the light source. However, a wavelength-based interferometer involves two main problems. The first one is the 2\( \pi \) ambiguity. As in the case of a He–Ne laser interferometer, we can only detect the phase of the interference signal. The phase detection problem involves the 2\( \pi \) ambiguity. The second one is the realization of a single wavelength.

Three schemes can be used to produce a single wavelength from an FOFC. The first scheme is to filter a single wavelength.
The distance between two reflecting surfaces to turn the reflected light on and off in order to distinguish different reflecting surfaces. This paper presents a new approach to solve this IA problem for an absolute and arbitrary long-distance measurement.

The IA problem is studied from the viewpoint of the symmetry with the phase unwrapping (PU) problem. The remainder of this paper is organized as follows. Section 2 briefly reviews the PU problem and the synthetic wavelength method, and then, it describes the IA problem and the proposed method. Section 3 presents the numeral experiments. Finally, Section 4 summarizes the main conclusions and future studies.

2 PRINCIPLES

2.1 Phase unwrapping problem and synthetic wavelength method

The PU problem, which has been extensively studied previously [19]–[21], is only briefly reviewed here to introduce basic concepts. Assuming \( h \) to be the distance between two points, the PU problem can be expressed as

\[
h = \lambda \varphi /2\pi = \lambda (\varphi_N + \varphi_\Delta) /2\pi
\]

(1)

Here, \( \lambda \) is the wavelength of the light source used for the measurement, and \( \varphi_N /2\pi \) and \( \varphi_\Delta /2\pi \) are the integral part (\( \varphi_N /2\pi \): integer) and the excess fractional part (\( 0 \leq \varphi_\Delta /2\pi < 1 \)) of the phase \( \varphi \), respectively.

By using the optical heterodyne measurement method [22], Fourier transform method [23], and fringe scanning method [24], we can measure the excess fractional part \( \varphi_\Delta \). By using the optical heterodyne measurement method, because the beat frequency of two different frequencies is low, an excess fractional part can be detected by using a phase meter. The other two methods find an excess fractional part by using the \( \tan^{-1} \) function. The excess fractional part can be calculated assuming that \( -2\pi \leq \varphi_\Delta < 2\pi \) holds. A phase ambiguity corresponding to the integral part of the phase remains. The PU problem is nothing but the problem of finding the true value \( \varphi \) from the excess fractional part \( \varphi_\Delta \), i.e., finding \( \varphi_N \).

Techniques for solving the PU problem using two or more wavelengths [25]–[30], called as the synthetic wavelength method, have already been introduced. We can understand this individual-point-based PU process as follows.

The distance \( h \) can be expressed as follows by using two wave-
lengths.

\[ h = \frac{\lambda_1 \varphi_1}{2\pi} = \frac{\lambda_1 (\varphi_{N1} + \varphi_{A1})}{2\pi} + \frac{\lambda_2 (\varphi_{N2} + \varphi_{A2})}{2\pi} \]  \quad (2)

Here, \( \lambda_1 \) and \( \lambda_2 \) are the two wavelengths. When \( h \) is measured by using different wavelength \( \lambda_i(i) \), \( \varphi_{N(i)} \) and \( \varphi_{A(i)} \) are the integral and the excess fractional parts of the phase \( \varphi_i \), respectively.

By using two wavelengths, the following synthetic wavelength is obtained:

\[ \lambda_{12} = \frac{\lambda_1 \lambda_2}{|\lambda_1 - \lambda_2|} \]  \quad (3)

Furthermore, \( h \) can be expressed as

\[ h = \frac{\lambda_{12} (\varphi_{N12} + \varphi_{A12})}{2\pi} \]  \quad (4)

For example, the synthetic frequency \( \lambda_{12} \) becomes 3.33 \( \mu \m \) when we assume \( \lambda_1 = 532 \text{ nm} \) and \( \lambda_2 = 633 \text{ nm} \). As described in previous studies \([19, 20]\), when we know that \( h < \lambda_{12} \) in advance, we can estimate the integral part \( \varphi_{N12} \) based on the two excess fractional parts. Then, the true value of the distance \( h \) can be unambiguously determined.

### 2.2 Integer ambiguity problem and synthetic APRIL method

When an arbitrary length \( h \) is assumed to be a function of APRIL \( AL \), it can be written as

\[ h = AL \times N = AL \times (N + \Delta) \]  \quad (5)

Here, \( N \) and \( \Delta \) are the integral and the excess fractional part, respectively.

In previous experiments, we showed that \( \Delta \) can be measured from the distance between the peaks of the envelopes of the observed interference fringes in a multi-pulse train interferometer \([31]\). When measuring \( h \) with a single APRIL, the IA problem occurs because we do not know the positional relationship between the two pulse trains that generate the cross-correlation function.

From the symmetry between Eq. (1) and Eq. (5), we propose the synthetic APRIL (S-APRIL) method in which two different APRILs are used to produce a synthetic APRIL to solve the IA problem.

The distance \( h \) measured by using two APRILs can be expressed as

\[ h = AL_1 \times (N_1 + \Delta_1) = AL_2 \times (N_2 + \Delta_2) \]  \quad (6)

Here, \( AL_1 \) and \( AL_2 (AL_2 < AL_1) \) are the two APRILs. \( N(i) \) and \( \Delta(i) \) are the integral and the excess fractional part of APRIL \( AL(i) \), respectively.

The S-APRIL \( AL_{12} \) is defined as

\[ AL_{12} = \frac{AL_1 AL_2}{|AL_1 - AL_2|} \]  \quad (7)

\( AL_{12} \) becomes 12 m when \( AL_1 = 4 \text{ m} \) and \( AL_2 = 3 \text{ m} \).
NUMERAL SIMULATIONS

Figure 3 shows the numeral simulation of the S-APRIL method. The object $x(n)$ measurement ($n$ is the sample index) is considered to be a sin wave with amplitude of 12 m, as shown in Figure 3(a). In Figure 3(a)–(c) and (e,f), the unit of the vertical axis is meters. The horizontal axis in Figure 3 indicates the sample index (arb. unit). The excess fractional parts $\Delta_1$ and $\Delta_2$ for APRILs for $AL_1 = 4$ m and $AL_2 = 3$ m are shown in Figure 3(b) and (c), respectively. The subtraction of the two excess fractional parts $\Delta_{12}$, as given in Eq. (9), is shown in Figure 3(d). In Figure 3(d), the unit of the vertical axis is arb. unit. Based on the value of $\Delta_{12}$, the measured length can be obtained from Eq. (11) as shown in Figure 3(e). To clarify the measurement accuracy, Figure 3(f) shows the difference between the object (Figure 3(a)) and the measured length (Figure 3(e)).

Figure 3(f) shows that we can accurately measure a length within 12 m and that if the value of the object is beyond the range of the synthesized APRIL, an error (peak in Figure 3(f) having an amplitude of 12 m) occurs.

Because the measured excess fractional parts may be noisy, the length measurement for a practical real-world application is actually quite challenging. We discuss this noisy case by using the same computer simulation to illustrate the effects of noise on the estimation process of an integral part. Suppose that we have the discrete signal $x(n)$, and then, we add white noise $wn(n)$ to this signal as follows:

$$x_{\text{noise}}(n) = x(n) + wn(n)$$

Then, we measure the noisy signal as

$$\Delta_{\text{noise}}(x) = I[x_{\text{noise}}(n)]$$

Here, $I(\cdot)$ is used to measure the excess fractional part. We assume that the two excess fractional parts share the same level but have different noise.

We follow the same process to solve the noisy IA problem. First, we set the variance of the noise to a value of $10^{-6}$ which means that we can measure a distance of 1 m with an accuracy of the order of micrometers. The result is shown in Figure 4. With the exception of Figure 4(a), each graph in Figure 4 shows the same calculation as that performed and shown in Figure 3. Figure 4(a) shows the two noisy signals (solid line and dashed line) and the difference between them (dotted line). The average value (without the value beyond the range of the S-APRIL) of the difference between the object and the measured length is $1.2 \times 10^{-6}$ m, and its variance is $3.6 \times 10^{-9}$ m. In this case, the noise does not adversely affect the estimation of the integral part.

The noise variance is now set to a higher value of $10^{-2}$ (Figure 5). We can observe that the higher noise level seriously affects the estimation of the integral part because many sig-
nals (with true values near 0 m or 12 m) exceed the unambiguous range of the measurement. Owing to the existence of a fake estimation, the estimation of the integral part of the signal, which is close to the measurement range, becomes quite challenging. The average value (without the value at which errors occur) of the difference between the object and the measured length is 0.008 m, and its variance is 0.08 m. We have found that in the case of a high level of noise, by selecting the unambiguous measurement range as being approximately two times of the target object, the S-APRIL method is feasible. For example, if the unambiguous measurement range is 12 m, length measurement in the range from 3 to 9 m is very stable.

4 CONCLUSIONS

We introduced the symmetry of the PU problem and the IA problem. When an arbitrary length was measured as a function of the APRIL, from this symmetry, we proposed the synthetic APRIL (S-APRIL) method to determine the integer part. As a synthetic wavelength method that can eliminate the ambiguity of $2\pi$ and determine an absolute and arbitrary short length (of the order of micrometers), S-APRIL can disambiguate the repeat interval and determine an absolute and arbitrary long length (of the order of meters). We confirmed the feasibility of the S-APRIL method and its measurement accuracy through a numerical simulation. The results of this study should contribute toward easy, highly accurate FOFC-based length measurements.

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